Aufgabe 10.1

$f_x(x,y) = 2x-2$ $\quad$ $f_{xx}(x,y) = 2$

$f_y(x,y) = 6y-9$ $\quad$ $f_{yy}(x,y) = 6$

$f_{xy}(x,y) = 0$

Notw. Bed.

$\text{I} \quad 0 = 2x-2 \iff x = 1$

$\text{II} \quad 0 = 6y-9 \iff y = 3/2$

Hinr. Bed.

$D(x,y) = 12 \underset{\text{immer}}{>} 0$

$f_{xx}(x,y) = 2 \underset{\text{immer}}{>} 0$

d.h. $(1; 3/2)$ glob. Min

Aufgabe 10.2

$$U(x_A, x_B) = p_A \cdot x_A + p_B \cdot x_B$$
$$= 48x_A - 4x_A^2 + 44x_B - 2x_B^2$$

$$K(x_A, x_B) = 100 + 8x_A + 12x_B$$

$$G(x_A, x_B) = -4x_A^2 - 2x_B^2 + 40x_A + 32x_B - 100$$

$$G_{x_A}(x_A, x_B) = -8x_A + 40 \qquad G_{x_A x_A}(x_A, x_B) = -8$$

$$G_{x_B}(x_A, x_B) = -4x_B + 32 \qquad G_{x_B x_B}(x_A, x_B) = -4$$

$$G_{x_A x_B}(x_A, x_B) = 0$$

Notw. Bed.

$$\text{I} \quad 0 = -8x_A + 40 \iff x_A = 5$$
$$\text{II} \quad 0 = -4x_B + 32 \iff x_B = 8$$

Hinr. Bed.

$D(x_A, x_B) = 32 \underset{\text{immer}}{>} 0$ und $G_{x_A x_A}(x_A, x_B) \underset{\text{immer}}{<} 0$

d.h. $(5; 8)$ glob. Max

$$p_A = 28, \quad p_B = 28$$

$$G(5; 8) = 128 \text{ GE}$$

Aufgabe 10.3

$f_{x_1}(x_1,x_2) = \frac{1}{2}x_1 + x_2 + 1$

$f_{x_2}(x_1,x_2) = x_2^2 + x_1 - 1$

$f_{x_1x_1}(x_1,x_2) = \frac{1}{2}$

$f_{x_2x_2}(x_1,x_2) = 2x_2$

$f_{x_1x_2}(x_1,x_2) = 1$

Notwendige Bed.

$\text{I} \quad 0 = \frac{1}{2}x_1 + x_2 + 1$

$\text{II} \quad 0 = x_1 + x_2^2 - 1$

$\text{I} - \frac{1}{2}\cdot\text{II} \quad 0 = x_2 - \frac{1}{2}x_2^2 + \frac{3}{2} \Leftrightarrow x_2 = 3$ od. $x_2 = -1$

1\. Fall: $x_2 = 3$ $\quad \text{I} \; 0 = \frac{1}{2}x_1 + 4 \Leftrightarrow x_1 = -8$

2\. Fall: $x_2 = -1$ $\quad \text{I} \; 0 = \frac{1}{2}x_1 \Leftrightarrow x_1 = 0$

d.h. $\binom{-8}{3}$ und $\binom{0}{-1}$ mögl. Extremstellen

Hinr. Bed.

$D(-8;3) = 2 > 0$ und $f_{x_1x_1}(x_1,x_2) = \frac{1}{2} \underset{\text{immer}}{>} 0$

d.h. $(-8;3)$ lok. Min

$D(0;-1) = -2 < 0$

d.h. $(0;-1)$ Sattelstelle

Aufgabe 10.4

$$G(x_1, x_2) = 50x_1 - 2x_1^2 + 100x_2 - 4x_2^2$$
$$- 10x_1 - 10x_2 - 2x_1x_2 - 100$$

$$G(x_1, x_2) = -2x_1^2 - 4x_2^2 - 2x_1x_2 + 40x_1$$
$$+ 90x_2 - 100$$

Notw. Bed.

$$\text{I} \quad 0 = G_{x_1}(x_1, x_2) = -4x_1 - 2x_2 + 40$$
$$\underline{\text{II}} \quad 0 = G_{x_2}(x_1, x_2) = -2x_1 - 8x_2 + 90$$

$$2 \cdot \underline{\text{II}} - \text{I} \quad 0 = -14x_2 + 140 \quad \Leftrightarrow \quad x_2 = 10$$

$$\text{I} \quad 0 = -4x_1 - 20 + 40 \quad \Leftrightarrow \quad x_1 = 5$$

Hinr. Bed.

$$D(x_1, x_2) = (-4) \cdot (-8) - (-2)^2 = 28 \underset{\text{immer}}{>} 0$$
$$G_{x_1x_1}(x_1, x_2) = -4 \underset{\text{immer}}{<} 0$$

d.h. $(5;10)$ glob. Max und $G(5;10) = 450$

$$p_1(5;10) = 40$$

$$p_2(5;10) = 60$$

Aufgabe 10.5

$K(x,y) = 2000 + 20x + 20y$

$U(x,y) = p_{USA} \cdot x + p_D \cdot y$

$= 100x - \frac{x^2}{2} + 120y - \frac{y^2}{4}$

a) $G(x,y) = -\frac{1}{2}x^2 - \frac{1}{4}y^2 + 80x + 100y - 2000$

b) $G_x(x,y) = -x + 80 = 0 \quad \Rightarrow x = 80$

$G_y(x,y) = -\frac{1}{2}y + 100 = 0 \quad \Rightarrow y = 200$

d.h. $(80;200)$ stationärer Punkt

c) $G_{xx}(x,y) = -1 \qquad G_{xy}(x,y) = 0$

$G_{yy}(x,y) = -\frac{1}{2}$

$D(x,y) = \frac{1}{2} \underset{\text{immer}}{>} 0$

$G_{xx}(x,y) = -1 \underset{\text{immer}}{<} 0$

d.h. $(80;200)$ glob. Max

d) $G(80;200) = 11\,200$ GE

Aufgabe 10.6

a) $U_1(p_1, p_2) = x_1 \cdot p_1$

$= 100p_1 - 2p_1^2 - p_1 p_2$

$U_2(p_1, p_2) = x_2 \cdot p_2$

$= 120p_2 - p_1 p_2 - 3p_2^2$

$K_1(p_1, p_2) = 120 + 2(100 - 2p_1 - p_2)$

$= 320 - 4p_1 - 2p_2$

$K_2(p_1, p_2) = 120 + 2(120 - p_1 - 3p_2)$

$= 360 - 2p_1 - 6p_2$

$G_1(p_1, p_2) = U_1(p_1, p_2) - K_1(p_1, p_2)$

$= -2p_1^2 - p_1 p_2 + 104p_1 + 2p_2 - 320$

$G_2(p_1, p_2) = U_2(p_1, p_2) - K_2(p_1, p_2)$

$= -3p_2^2 - p_1 p_2 + 126p_2 + 2p_1 - 360$

$G(p_1 p_2) = -2p_1^2 - 2p_1 p_2 + 106p_1 - 3p_2^2 + 128p_2 - 680$

Aufgabe 10.6

b) $G_{p_1}(p_1, p_2) = -4p_1 - 2p_2 + 106$

$G_{p_2}(p_1, p_2) = -2p_1 - 6p_2 + 128$

Notw. Bed.

$\text{I} \quad 0 = -4p_1 - 2p_2 + 106$

$\text{II} \quad 0 = -2p_1 - 6p_2 + 128$

$\text{I} - 2 \cdot \text{II} \quad 0 = 10p_2 - 150 \Leftrightarrow p_2 = 15$

$p_1 = 19$

d.h. $(19; 15)$ stationärer Punkt

Hinr. Bed.

$G_{p_1p_1}(p_1, p_2) = -4$ $\quad G_{p_1p_2}(p_1, p_2) = -2$

$G_{p_2p_2}(p_1, p_2) = -6$

$D(p_1, p_2) = 20 \underset{\text{immer}}{>} 0$

$G_{p_1p_1}(p_1, p_2) = -4 \underset{\text{immer}}{<} 0$

d.h. $(19; 15)$ glob. Max

$G(19; 15) = 1287$ GE

Aufgabe 10.6

c) $p_2 = 16$

$x_1 = 100 - 2p_1 - 16 = 84 - 2p_1$

$U_1(p_1) = p_1 \cdot x_1 = 84p_1 - 2p_1^2$

$K_1(p_1) = 120 + 2(84 - 2p_1) = 288 - 4p_1$

$G_1(p_1) = 88p_1 - 2p_1^2 - 288$

$G_1'(p_1) = 88 - 4p_1$ $\qquad$ $G_1''(p_1) = -4$

Notw. Bed.

$0 = 88 - 4p_1 \iff p_1 = 22$

Hinr. Bed.

$G_1''(p_1) = -4 \underset{\text{immer}}{<} 0$

d.h. $p_1 = 22$ glob. Max

d) ohne Streit: $p_1 = 19$; $p_2 = 15$

mit Streit: $p_1 = 22$; $p_2 = 16$

d.h. Streit beilegen